

# MATH 355 Calculus III

## Lab #1 - Polar Limaçons

- In this lab we will extend the polar plot of the limaçon given in class. This class of polar graphs is called the polar limaçon. The polar limaçon is defined by

$$r(\theta) = a \pm b\sin(c\theta) \text{ or } r(\theta) = a \pm b\cos(c\theta)$$

Please note that the limaçon we studied in class is a special case of the polar limaçon where  $c = 1$ .

- Let's first review the graph of the limaçon by defining  $r(\theta)$  and plotting:

```
r[θ_] := 1 + Sin[θ]
PolarPlot[r[θ], {θ, 0, 2 π}]
```

- Recall that we may add color to a graph and combine several on one set of axes:

```
r1[θ_] := 1 + 2 Cos[θ]
r2[θ_] := 1 - 2 Cos[θ]
r3[θ_] := 1 + 2 Sin[θ]
r4[θ_] := 1 - 2 Sin[θ]
p1 = PolarPlot[r1[θ], {θ, 0, 2 π}, PlotStyle → RGBColor[1, 0, 0], PlotRange → {{-3, 3}, {-3, 3}}]
p2 = PolarPlot[r2[θ], {θ, 0, 2 π}, PlotStyle → RGBColor[0, 1, 0]]
p3 = PolarPlot[r3[θ], {θ, 0, 2 π}, PlotStyle → RGBColor[0, 0, 1]]
p4 = PolarPlot[r4[θ], {θ, 0, 2 π}, PlotStyle → RGBColor[1, 1, 0]]
Show[p1, p2, p3, p4]
```

- Now let's introduce the **Manipulate** function to change the  $a$  parameter in our original definition of the polar limaçon. Recall that **PlotRange** forces *Mathematica* to show the plot over the range given in **PlotRange**. This is critical to plotting with **Manipulate** as it keeps all the graphs the same size. We will keep  $c = 1$  for the time being.

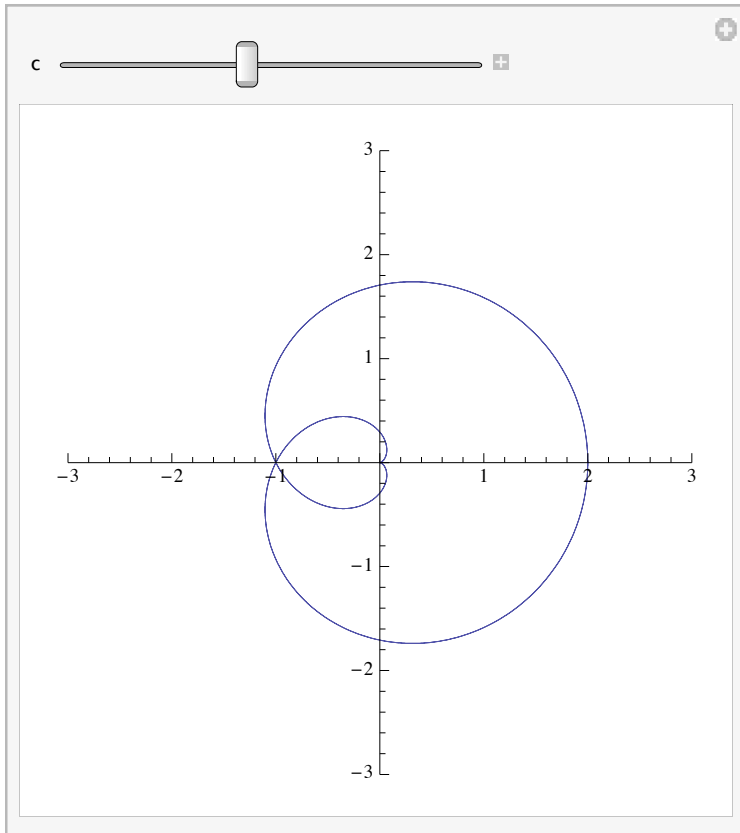
```
Manipulate[PolarPlot[a + Cos[θ], {θ, 0, 2 π}, PlotRange → {{-3, 3}, {-3, 3}}], {a, -2, 2}]
```

- More options are given for **Manipulate** by clicking the **+** icons within the output window. Clicking on the **+** inside the square next to the slider enables you to play through the values. Next let's vary the  $b$  parameter in a similar way:

```
Manipulate[PolarPlot[1 + b * Cos[θ], {θ, 0, 2 π}, PlotRange → {{-3, 3}, {-3, 3}}], {b, -2, 2}]
```

- These are all examples of the limaçons that we looked at in class. Let's generalize to polar limaçons where  $c \neq 1$ , but keep  $a = b = 1$ . We will increase  $\theta$  to  $8\pi$  and  $c$  from  $-4$  to  $4$  in order to see additional changes.

```
Manipulate[PolarPlot[1 + Cos[c *  $\theta$ ], { $\theta$ , 0, 8  $\pi$ }, PlotRange -> {{-3, 3}, {-3, 3}}, {c, -4, 4}]
```



- As you can see, these are complicated and varied plots that vary considerably from  $c = -4$  to  $4$ .

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## Lab #1:

- You are to classify the polar limaçons obtained as  $c$  varies from  $-4$  to  $4$ , keeping  $a = b = 1$ .
- If you wish to explore further, try changing the ratio  $\frac{a}{b}$  from  $1$  to something else.