Calculus III
Lab #4: Tangent Planes

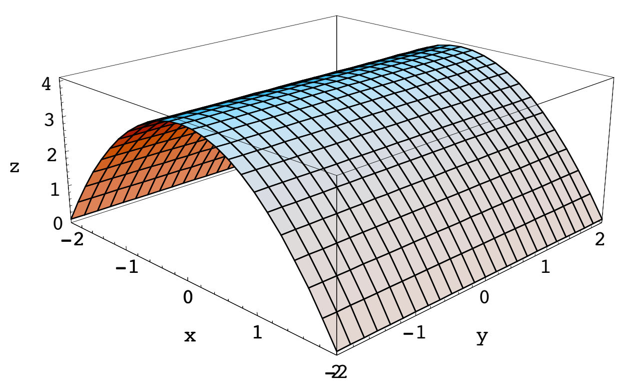
- **In this lab we will be graphing surfaces of the form z = f(x, y) at the point (x_0, y_0, z_0) on the surface. In order to do this we will need the partial derivatives of f(x, y) evaluated at the point (x_0, y_0). The usual equation for this tangent plane is:**

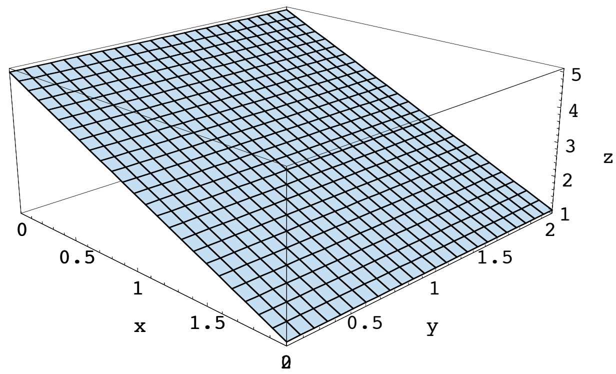
- $$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0$$

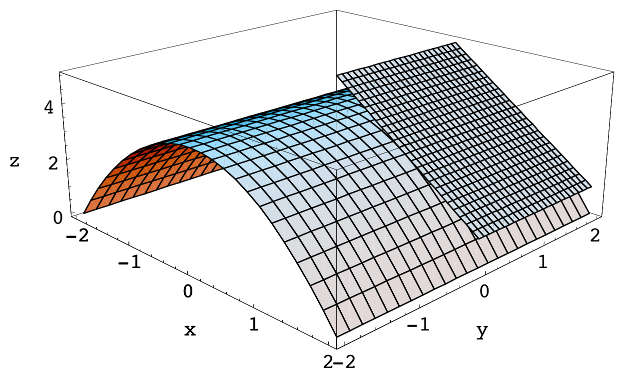
- **We will solve for z and obtain the form $z = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) + z_0$**

- **One way to get the partial derivative of f(x, y) with respect to x is D[f[x, y], x]. We can append to this the assignment x = x_0 and y = y_0. To do this use the notation: /.{ x → x_0, y → y_0 }.**
  Thus given the surface f(x, y) = 4 - x^2 and the point (1, 1, 3), the tangent plane to the surface is defined as follows:

```
f[x_, y_] := 4 - x^2;
x0 = 1;
y0 = 1;
z0 = f[x0, y0];
dx = D[f[ x, y ], x] /. {x → x0, y → y0};
dy = D[f[ x, y ], y] /. {x → x0, y → y0};
p[x_, y_] := dx * (x - x0) + dy * (y - y0) + z0;
p1 =
  Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, ViewPoint → {2, -2, 1}, AxesLabel → {x, y, z}];
p2 = Plot3D[p[x, y], {x, 0, 2}, {y, 0, 2}, ViewPoint → {2, -2, 1}, AxesLabel → {x, y, z}];
Show[p1, p2];
```







- **Exercises**

- **1. Graph the surface $z = 4 - x^2 - y^2$ and its tangent plane at (1, 1, 2).**

- **2. Graph the surface $z = 2\sin(x \cdot y)$ and its tangent plane at ($\frac{\pi}{6}$,1, 1). Change viewpoint for best view.**

- **3. Define your own surface. Plot it and its tangent plane at a point.**