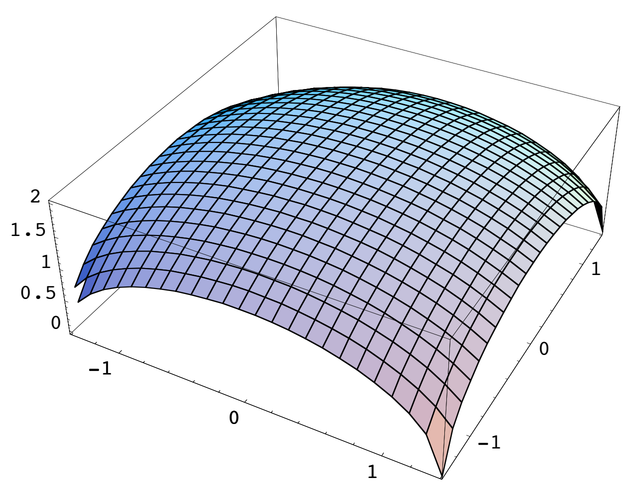
Calculus III
Lab #5: Triple Integrals in Rectangular, Cylindrical and Spherical Coordinates

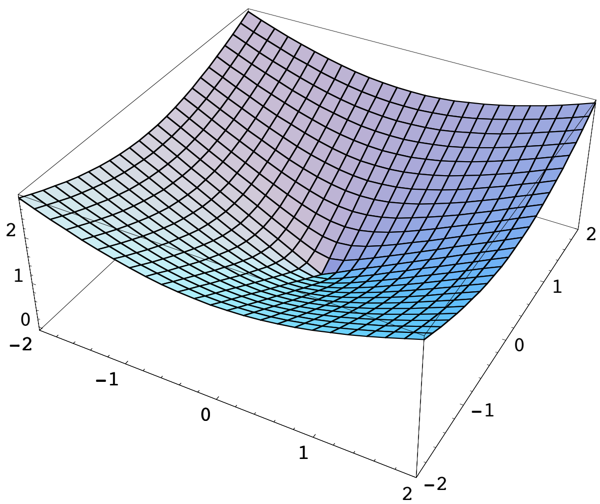
- **In this lab you are to plot 2 surfaces and determine the volume of the solid bounded by them by setting up triple integrals rectangular, cylindrical and spherical coordinates.**

- **Let's plot each of the surfaces given below, a sphere and a cone, and determine the volume of the solid bounded by them.**

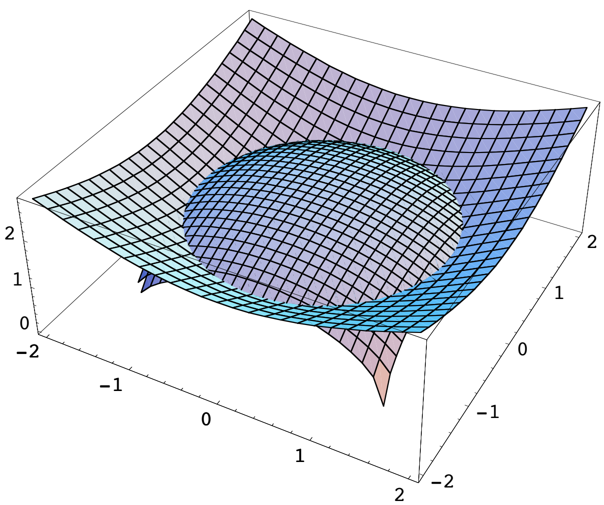
```
Clear[f, g, x, y];
f[x_, y_] := √(4 - x² - y²);
g[x_, y_] := √(x² + y²);

p1 = Plot3D[f[x, y], {x, -√2, √2}, {y, -√2, √2}];
p2 = Plot3D[g[x, y], {x, -2.0, 2.0}, {y, -2.0, 2.0}];

Show[p1, p2];
```

```
Plot3D::gval : Function value 0. + 2.98023×10^-8 i at grid point xi = 1, yi = 1 is not a real number. More…
```






```
Graphics3D::nlist3 : {-1.41421, -1.41421, 0. + 2.98023×10^-8 i} is not a list of three numbers. More…
```



- **There is some protest here by *Mathematica* at the endpoint ($-\sqrt{2}$, $-\sqrt{2}$) because of roundoff that leads to the square root of a negative number..**

- **The volume bounded by the two surfaces can be found by evaluating any of the three triple integrals. The N or numeric option is used since the first integral gives *Mathematica* a little trouble.**

$$\text{v1} = 4\,\text{N}\left[\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 1\,dz\,dy\,dx,\ 4\right]$$

$$\text{v2} = 4\,\text{N}\left[\int_0^{\frac{\pi}{2}}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta,\ 4\right]$$

$$\text{v3} = 4\,\text{N}\left[\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{4}}\int_0^{2} \rho^2 * \text{Sin}[\phi]\,d\rho\,d\phi\,d\theta,\ 4\right]$$

```
4.907

4.907

4.907
```

- **Exercises**

- **1. Plot the the plane 6x + 4y + 5z = 60 and find its volume in the first octant with rectangular coordinates.**

- **2. Plot the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ and find the volume bounded with cylindrical coordinates.**

- **3. Plot the surfaces $x^2 + y^2 + z^2 = 16$ and $z = 2$ and find the volume bounded with spherical coordinates.**