# Calculus III
# Lab #6: Work in Vector Field

- **In this lab we will find the work done in a vector field from ( -1, -1 ) to ( 1, -1 ) to ( 1, 1 ) to ( -1, 1 ) to ( -1, -1 ). Let us suppose the vector field is $\vec{F}$( x, y ) = < $x^2 + y^3$ , xy >.**

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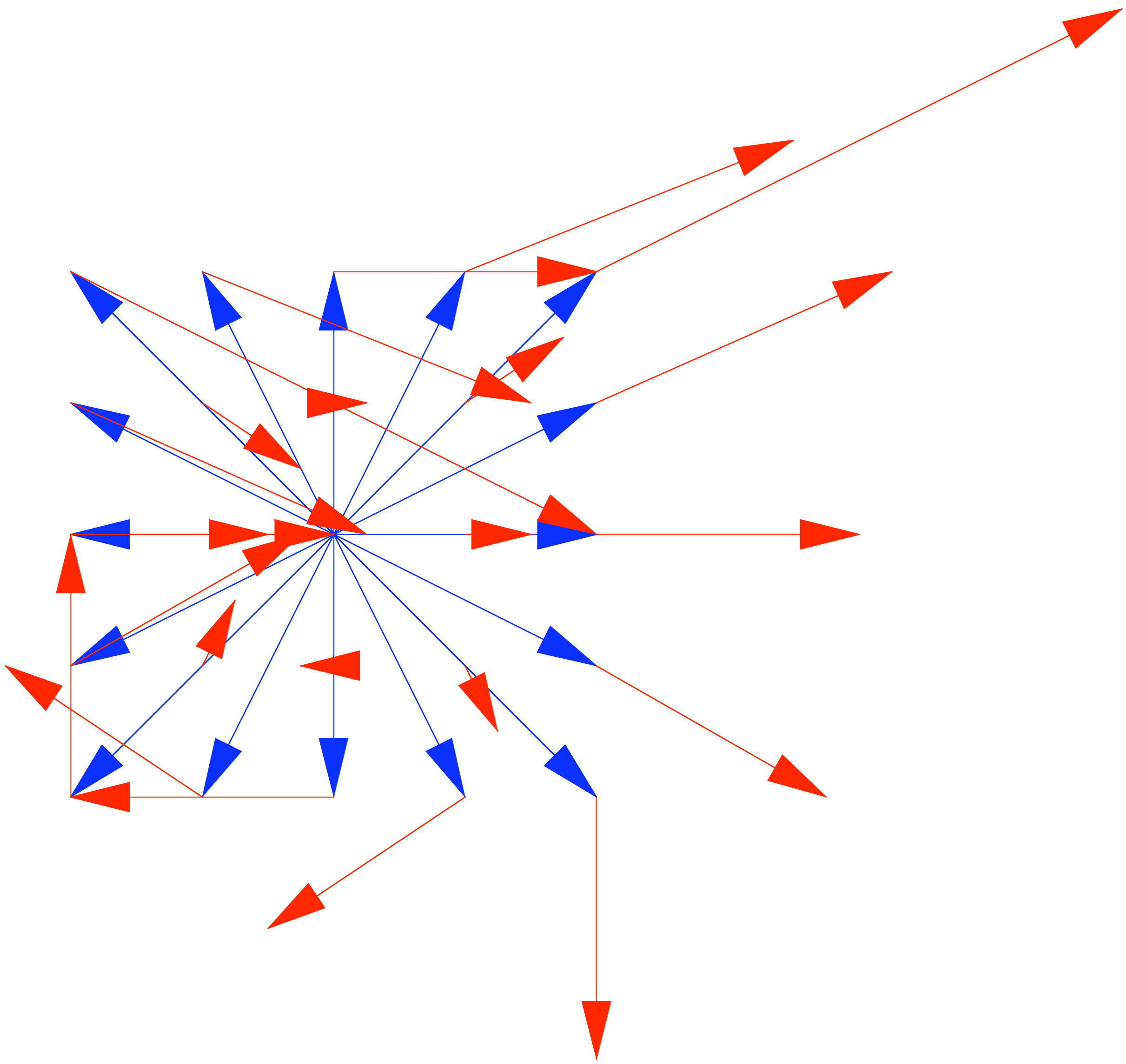
Needs["Graphics`Arrow`"]
```

```
Clear[f, g, x, y];
f[x_, y_] := x^2 + y^3;
g[x_, y_] := x * y;
force = Graphics[{RGBColor[1, 0, 0],
    Table[Arrow[{i, j}, {i + f[i, j], j + g[i, j]}], {i, -1, 1, .5}, {j, -1, 1, .5}]}];
path1 = Graphics[{RGBColor[0, 0, 1], Table[Arrow[{0, 0}, {t, -1}], {t, -1, 1, .5}]}];
path2 = Graphics[{RGBColor[0, 0, 1], Table[Arrow[{0, 0}, {1, t}], {t, -1, 1, .5}]}];
path3 = Graphics[{RGBColor[0, 0, 1], Table[Arrow[{0, 0}, {-t, 1}], {t, -1, 1, .5}]}];
path4 = Graphics[{RGBColor[0, 0, 1], Table[Arrow[{0, 0}, {-1, -t}], {t, -1, 1, .5}]}];
Show[path1, path2, path3, path4, force, AspectRatio → Automatic];
```



- **Now let's find the work done along each side of the square and add them to find the total work done. Then we will verify this result by applying Green's Theorem.**

```
w1 = ∫_{-1}^{1} {t^2 - 1, -t}.{1, 0} ⅆt;
w2 = ∫_{-1}^{1} {1 + t^3, t}.{0, 1} ⅆt;
w3 = ∫_{-1}^{1} {t^2 + 1, -t}.{-1, 0} ⅆt;
w4 = ∫_{-1}^{1} { 1 - t^3, t}.{0, -1} ⅆt;
wTotal = w1 + w2 + w3 + w4
g = ∫_{-1}^{1} ∫_{-1}^{1} (D[g[x, y], x] - D[f[x, y], y]) ⅆy ⅆx
```

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- **Exercises**

- **Define a non-conservative vector field in the plane. Find the work done along the circular path $\vec{r}(t) =$ < Cos( $t$ ), Sin( $t$ )> two ways. Plot the vector field and the path.**